UNBC UNIVERSITY OF NORTHERN BRITISH COLUMBIA
Department of Mathematics & Statistics

**Winter 2024, Math 204, HW Assignment**
Due: January 31, 2024, in class.

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The numbers between brackets in the margin represent the marks assigned to the question. The maximum grade is 100.

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(15) 1. Give a formula $F(x,y) = M(x,y)\mathbf{i} + N(x,y)\mathbf{j}$ for the vector field in a plane that has the properties that $F(0,0) = \vec{0}$ and that at any other point $(a,b)$, $F$ is tangent to circle $x^2+y^2 = a^2+b^2$ and points in the clockwise direction with magnitude $|F| = \sqrt{a^2+b^2}$.

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2. Assume that an electric field in the $xy$-plane caused by an infinite line of charge along the $x$-axis is a gradient field with potential function

$$V(x,y) = c\log\left(\frac{r_0}{\sqrt{x^2+y^2}}\right),$$

where $c > 0$ is a constant and $r_0$ is a reference distance at which the potential is assumed to be zero.

(5) (a) Find the components of the electric field in the $x$- and $y$-directions, where $\mathbf{E}(x,y) = -\nabla V(x,y)$.

(5) (b) Show that the electric field at a point in the $xy$-plane is directed outward from the origin and has magnitude $|\mathbf{E}| = \frac{c}{r}$, where $r = \sqrt{x^2+y^2}$.

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3. A flow line (or streamline) of a vector field $F$ is a curve $\mathbf{r}(t)$ such that $d\mathbf{r}/dt = F(\mathbf{r}(t))$. Note that if F represents the velocity field of a moving particle, then the flow lines are paths taken by the particle. Therefore, flow lines are tangent to the vector field. For the following exercises, show that the given curve $\mathbf{c}(t)$ is a flow line of the given velocity vector field $\mathbf{F}(x,y,z)$.

(5) (a) $\mathbf{c}(t) = (e^{2t}, \log t, \frac{1}{t})$; $\mathbf{F}(x,y,z) = (2x, z, -z^2)$.

(5) (b) $\mathbf{c}(t) = (\sin t, \cos t, e^t)$; $\mathbf{F}(x,y,z) = (y, -x, z)$.

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4. Compute

(5) (a) $\int_\gamma (y^2 - xy)\, dx$ where $\gamma$ is the part of the curve $x = e^y$ from $(1,0)$ to $(e,1)$.

(5) (b) $\int_\gamma xy^4\, ds$ where $\gamma$ is the right half of the circle $x^2 + y^2 = 25$.

(5) (c) $\int_\gamma xy^2\, ds$ where $\gamma$ is a triangle with vertices $(0,1,2)$, $(1,0,3)$, and $(0,-1,0)$.

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(5) 5. Compute the flux of $\mathbf{F} = x^2\mathbf{i} + y\mathbf{j}$ across a line segment from $(0,0)$ to $(1,2)$.

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(10) 6. An object moves in force field $\mathbf{F}(x,y,z) = y\mathbf{i} + 2(x+1)y\mathbf{j}$ counterclockwise from point $(2,0)$ along elliptical path $x^2 + 4y^2 = 4$ to $(-2,0)$, and back to point $(2,0)$ along the $x$-axis. How much work is done by the force field on the object?

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(10) 7. Justify (or verify the validity of) the Fundamental Theorem of Line Integrals for $\int_C F \cdot dr$ in the case when $F(x,y) == (2x+2y)\mathbf{i} + (2x+2y)\mathbf{j}$ and $C$ is a portion of the positively oriented circle $x^2 + y^2 = 25$ from $(5,0)$ to $(3,4)$.

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(15) 8. Find the value of

$$\int_C \left(\arctan\frac{y}{x} - \frac{xy}{x^2+y^2}\right) dx + \left(\frac{x^2}{x^2+y^2} + e^{-y}(1-y)\right) dy$$

where $C$ is **any** smooth curve from $(1,1)$ to $(-1,2)$.

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(10) 9. Let $F(x,y,z) = x^2\, i + z\sin(yz)\, j + y\sin(yz)\, k$. Calculate $\int_C F \cdot dr$, where $C$ is a path from $A(0,0,1)$ to $B(3,1,2)$.

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**TOTAL MARKS: 100**