MATH 100, Tutorial 6
(Week of February 26, 2024)

Exercise 1 *State the Inverse Function Theorem.*

Exercise 2 *Use the given values to find* $(f^{-1})'(a)$*:*

(a) $f(\pi) = 0, \quad f'(\pi) = -1, \quad a = 0.$

(b) $f(\sqrt{3}) = \dfrac{1}{2}, \quad f'(\sqrt{3}) = \dfrac{2}{3}, \quad a = \dfrac{1}{2}.$

Exercise 3 *Let* $f(x) = \dfrac{2x+5}{x+3}$ *be a function defined on* $(-3, +\infty)$*. Also, let* g *be the function defined on* $(-\infty, 2)$ *by* $g(x) = \dfrac{5-3x}{x-2}$.

(a) Show that g *is the inverse function of* f.

(b) Express $\dfrac{d}{dx}(g(f(x)))$ *in terms of* g' *and* f'.

(c) Compute $g'(x)$ *in terms of* x *and deduce* $g'(f(x))$ *in terms of* x.

(d) Compute $f'(x)$ *in terms of* x.

Exercise 4 *Use the Inverse Function Theorem to show that* $\dfrac{d}{dx}(sin^{-1}(x)) = \dfrac{1}{\sqrt{1-x^2}}$.

Exercise 5 *Find* $\dfrac{dy}{dx}$ *for the given function:*

(a) $y = \arccos(\sqrt{x})$

(b) $y = \arcsin(x^2)$

(c) $y = \left(1 + \tan^{-1} x\right)^3$.