UNBC UNIVERSITY OF NORTHERN BRITISH COLUMBIA

Department of Mathematics & Statistics

Winter 2024, Math 100, HW Assignment 2

Due: February 2, 2024, in class.

The numbers between brackets in the margin represent the marks assigned to the question. The maximum grade is 100.

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1. Compute the following limits, if they exist (giving the necessary justifications). If the limit does not exist, state that.

(10) (a) $\lim_{x\to 7} \dfrac{4(x^2-8x+7)}{2x-14}$.

(10) (b) $\lim_{x\to 7} \dfrac{4(x-7)}{\sqrt{x}-\sqrt{7}}$.

(10) (c) $\lim_{x\to+\infty} \dfrac{4x^3-2\sqrt{x}}{\sqrt{x}+16x^3-x}$.

(10) (d) $\lim_{x\to 2} \dfrac{x^3-8}{x^2-4}$.

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2. For the following functions, sketch the graph **and** use the definition of a derivative to show that the function is not differentiable at $x = 1$.

(10) (a) $f(x) = \begin{cases} 2\sqrt{x}, & 0 \le x \le 1 \\ 3x-1, & x > 1. \end{cases}$

(10) (b) $f(x) = \begin{cases} -x^2+2, & x \le 1 \\ x, & x > 1. \end{cases}$

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(20) 3. Sketch the graph of one function $y = f(x)$ that simultaneously satisfies **all** of the following properties:

(a) $f'(x) < 0$ for $-2 \le x < 1$

(b) $f'(2) = 0$

(c) $f'(x) < 0$ for $x > 2$

(d) $f(2) = -4$ and $f(0) = -2$

(e) $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to +\infty} f(x) = -\infty$

(f) $f'(1)$ does not exist.

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4. Each of the limits in the list below represents the derivative of a function $y = f(x)$ at $x = a$. In each case, find $f(x)$ and a:

(10) (a) $\lim_{h \to 0} \dfrac{(2(2+h)^3 - (2+h)) - 14}{h}$.

(10) (b) $\lim_{h \to 0} \dfrac{(8+h)^{2/3} - 4}{h}$.

TOTAL MARKS: 100