Submission of your homework assignment will be via Blackboard. Please scan all your work into one single PDF file with multiple pages and upload the file on Blackboard (learn.unbc.ca). Late assignments will be stamped "late" by the system. Early submission of your homework is possible/accepted (online). PDF is the only accepted format. A smartphone can do high quality scans of your written solutions. There is no need to use a photocopy service to scan your homework. The important is that your writing is readable.

Problem 1.

Let $g : \mathbb{R} \to \mathbb{R}$. Find a function H such that for all x,

$$\int_{-\infty}^{x} g(t)dt = (H * g)(x).$$

Problem 2.

For $a \in \mathbb{R}$, let

$$f_a(t) := \begin{cases} 0, & t < a \\ 1, & t \geq a \end{cases} \qquad g_a(t) := \begin{cases} 0, & t < 0 \\ \exp(-at), & t \geq 0. \end{cases}$$

(a) Compute $f_a * f_b$ for $a, b \in \mathbb{R}$. Then, find $f_a * f_{-a}(x)$.

(b) Compute $g_a * g_b$.

Problem 3.

Suppose that

$$f * \exp(-x^2/2) = (1/2)x \exp(-x^2/4).$$

Find f.

Problem 4.

What is the convolution of a Gaussian kernel e^{-x^2} with itself? Hint: Use the Fourier transform.

Problem 5.

Find the inverse Fourier transform of the following functions:

$$\text{(a) } k, \qquad \text{(b) } ke^{-k^2}, \qquad \text{(c) } \frac{k}{(1+k^2)^2}.$$

Problem 6.

Use the Fourier transform to find a bounded solution to the differential equation

$$u'''' + u = e^{-2|x|}.$$

Problem 7.

In this exercise, we explain how convolution can be used to smooth out rough data. Let

$$g_\varepsilon(x); = \frac{\varepsilon}{\pi(\varepsilon^2 + x^2)}.$$

(a) If $f(x)$ is any (reasonable) function, show that $f_\varepsilon(x) = g_\varepsilon * f(x)$ for $\varepsilon \neq 0$ is a C^∞ function.

(b) Show that $\lim_{\varepsilon \to 0} f_\varepsilon(x) = f(x)$.