Topological and Normed Linear Spaces - MATH 402/602

(CRN 10773)
Department of Mathematics & Statistics,
University of Northern British Columbia

January 2022, MWF, 4:30 pm — 5:20 pm, 5–153

Essentials

Instructor:	Dr. Mohammad El Smaily
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Tel:	250-960-6624
Office:	T&L, 10-2044
Office Hours:	Monday and Wednesday 9:30 AM-10:30 AM.
	or by appointment (email)
Note:	office hours are held online due to COVID-19

Lectures:

Mon-Wed-Fri 4:30 pm — 5:20 pm, 5–153 (Library building)

Course Description:

This course introduces analysis on a metric space, normed vector space or a Hilbert space. Topics include: analysis on metric spaces, fixed point theorems, bounded linear operators between normed vector spaces, Banach Spaces, Hilbert Spaces, Fourier Transform, Compact Linear Maps and their Spectra, and criteria for compactness of a linear operator.

Prerequisites: MATH 302 Minimum Grade of C^{-}.

Textbook:

Functional Analysis by Peter D. Lax, Wiley-Interscience, New York, 2002, xix+ 580 pp.

Course Web Pages:

- We will use Blackboard https://learn.unbc.ca to post all materials for this course and make announcements.

Practice Problems

Each section in this book is followed by several exercises and problems which vary from easy to challenging. Feel free to work out the exercises which are relevant to each lecture on your own. I will solve some of these exercises during the semester. And will assign some others as homework. Remember that the homework is mainly to assess your understanding of the material and prepare you for the exams as well.

Grading Scheme:

Your Math 402/602 final grade is computed according to the following scheme:

Assignments: **30%**
Midterm Exam: **30%**
Final Exam: **40%**

Assignments:

Assignments will be given/posted on Blackboard and will be **handed in online–Blackboard**. There will be approximately 6 class assignments.
The due dates will be written on the assignments. Assignments must be handed in on time. Late assignments will only be accepted for medical or compassionate reasons.

Midterm Exams:

There will be **one midterm exam** on **Friday March 4**. If you have an unavoidable conflict with a scheduled exam, it is your responsibility to inform me as soon as possible (preferably one week in advance); decisions in this regard will be made on a case-by-case basis.

Final Exam:

TBA by the Registrar's Office. Final exam will be **comprehensive and will include all the material covered in the course**.

Important Dates:

First day of classes: Wed 5 January
Midterm 1: Friday, March 4
Add/Drop Date: Wednesday January 19
Family Day: Mon 21 February
Mid-Semester Break: Tuesday 22 February - Friday 25 February
Withdrawal Date: Thursday 24 February
Final Exam Period: Fri 8 April - Fri 22 April

Tentative syllabus

- 1. Linear Spaces: Axioms for linear spaces–Infinite-dimensional examples–Subspace, linear span–Quotient space–Isomorphism–Convex sets–Extreme subsets
- 5. Normed Linear Spaces
  - 5.1 Norms Norms for quotient spaces-Complete normed linear spaces- The spaces C, B- L^p spaces and Hölder's inequality
  - 5.2 Noncompactness of the unit ball
- 6. Hilbert Space

  - 6.1 Scalar product, Schwarz inequality-Parallelogram identity–Completeness
  - 6.2 Closest point in a closed convex subset, Orthogonal complement of a subspace-Orthogonal decomposition
  - 6.3 Linear functionals, Lax-Milgram lemma
  - 6.4 Linear span Orthogonal projection-Orthonormal bases, Gram-Schmidt process-Isometries of a Hilbert space
- 7. Applications of Hilbert Space Results
  - 7.2 Dirichlet's problem, Use of the Riesz-Frechet theorem – Use of the Lax-Milgram theorem–Use of orthogonal decomposition
- 8. Duals of Normed Linear Spaces
  - 8.1 Bounded linear functionals, Dual space
- 15. Bounded Linear Maps
  - 15.1 Boundedness and continuity Norm of a bounded linear map–Transpose
  - 15.4 Composition of bounded maps
- 16. Examples of Bounded Linear Maps
  - 16.1 Boundedness of integral operators, Integral operators of Hilbert-Schmidt type
  - 16.3 Examples of bounded integral operators, The Fourier transform, Parseval's theorem–The Laplace transform
- 20. Examples of Operators and Their Spectra
  - 20.1 Invertible maps, Boundary points of the spectrum
  - 20.2 Shifts,
  - 20.3 Volterra integral operators,
  - 20.4 The Fourier transform
- 21. Compact Maps
  - 21.1 Basic properties of compact maps Compact maps form a two-sided ideal
  - 21.2 The spectral theory of compact maps, The transpose of a compact operator is compact-The Fredholm alternative Historical note,
- 22. Examples of Compact Operators
  - 22.1 Compactness criteria Arzela-Ascoli compactness criterion-Rellich compactness criterion
  - 22.2 Integral operators, Hilbert-Schmidt operators
  - 22.3 The inverse of elliptic partial differential operators
  - 22.4 Operators defined by parabolic equations
  - 22.5 Almost orthogonal bases

Special Arrangements:

Students with disabilities who would like to receive access and academic accommodations through the Access Resource Centre (ARC) need to self-identify and register with the centre. Please see http://www.unbc.ca/access-resource-centre. The students who have registered for accommodations with the ARC must ensure that the instructor is informed of the necessary arrangements as soon as possible.

Academic Regulations:

It is the students' responsibility to familiarize themselves with the regulations concerning academic integrity and ensure that their course work conform to the principles of academic integrity. Please read the academic regulations found at:

http://www.unbc.ca/calendar/undergraduate/general/regulations.html.
In particular, read sections 40, 41, 42, 43, 44, and 45.

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