Submission of your homework assignment will be via Blackboard. Please scan all your work into one single PDF file with multiple pages and upload the file on Blackboard (learn.unbc.ca). Late assignments will be stamped "late" by the system. Early submission of your homework is possible/accepted (online). PDF is the only accepted format. A smartphone can do high quality scans of your written solutions. There is no need to use a photocopy service to scan your homework. The important is that your writing is readable.

Problem 1.

Let $(X, \|\cdot\|)$ be a normed vector space. This particularly implies that the triangle inequality (or subadditivity of $\|\cdot\|$) holds in $(X, \|\cdot\|)$. That is,

$$\forall x, y \in X, \quad \|x+y\| \leq \|x\| + \|y\|. \tag{1}$$

We start with the following

Definition 1 (Strict subadditivity)**.** *A norm is called strictly subadditive if in* (1)*, strict inequality holds except when x or y is a nonnegative multiple of the other.*

Show that the sup norms of $(l^\infty, \|\cdot\|_\infty)$ and $C([0,1])$ are not strictly subadditive.
Recall that for $f \in C([0,1])$, $\|f\|_\infty := \max_{x\in[0,1]} |f(x)|$.

Problem 2.

Is the set

$$\mathcal{U} := \{f \in C([0,1]) \ : \ |f(t)| < 1 \text{ for all } t \in [0,1]\}$$

open in $C[0,1]$ (equipped with the sup-norm)? Explain.

Problem 3.

Let $f_n(t) = t^n$. Obviously $f_n \in C([0,1])$ for all $n \in \mathbb{N}$. show that $f_n \longrightarrow 0$ in $(C([0,1]), \|\cdot\|_1)$ while $\{f_n\}_n$ does not converge (to any limit) in $(C([0,1]), \|\cdot\|_\infty)$.

Problem 4 (On equivalent norms)**.**

Show that the norms $\|\cdot\|_1$ and $\|\cdot\|_\infty$ are **not** equivalent on $C([a,b])$.